

Advanced Math

6-5

(Day 2)

DeMoivre's Theorem

DeMoivre's Theorem :

if z is a complex number and n is a positive integer, then

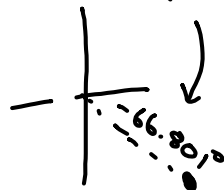
$$z^n = r^n [\cos(n\theta) + i \sin(n\theta)]$$

power of modulus \leftarrow \leftarrow power times the argument

$$*1) (3 - 7i)^5 = (\sqrt{58})^5 [\cos(-334.007^\circ) + i \sin(-334.007^\circ)]$$

$$\text{modulus} = \sqrt{3^2 + (-7)^2} = \sqrt{58}$$

$$\text{argument} = \tan^{-1}\left(\frac{-7}{3}\right) = -66.801^\circ$$



Multiply by the power,
in this case 5.

n th Roots of Complex Numbers :

For a positive integer n , the complex number $z = r(\cos\theta + i \sin\theta)$ has exactly n distinct roots given by

$$\sqrt[n]{r} \left(\cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n} \right)$$

where $k = 0, 1, 2, \dots, n - 1$

I never really use this formula. I know that there will be the same number of roots as the index. I divide the angle found with arctan() by the index. Then I add $360^\circ/n$ (index) and keep adding this angle until all the roots are found. (See next slide.)

Find the following root:

*2) $\sqrt[4]{3-7i}$

mod: $\sqrt{58}$

$\theta = -66.801^\circ$

4^{th} root, separated by 90°

$360^\circ/4 = 90^\circ$

root 1) $\sqrt[4]{\sqrt{58}} (\cos(-66.8^\circ) + i \sin(-66.8^\circ)) = .47 + 1.59i$

2) $\sqrt[4]{\sqrt{58}} (\cos 23.2^\circ + i \sin 23.2^\circ) = -1.59 + .47i$

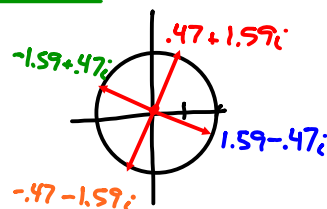
3) $\sqrt[4]{\sqrt{58}} (\cos 113.2^\circ + i \sin 113.2^\circ) = -.47 - 1.59i$

4) $\sqrt[4]{\sqrt{58}} (\cos 203.2^\circ + i \sin 203.2^\circ) = 1.59 - .47i$

Multiply these out to get the standard form.

I know that there will be the same number of roots as the index. I divide the angle found with arctan() by the index. Then I add $360^\circ/n$ (index) and keep adding this angle until all the roots are found.

$\sqrt[4]{\sqrt{58}} = (\sqrt{58}^{\frac{1}{2}})^{\frac{1}{4}} = \sqrt{58}^{\frac{1}{8}} = \sqrt[8]{58}$



Assignment:
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70-80 even,
89-100 all.